

Quantum knowledge, quantum belief, quantum reality

Notes of a QBist fellow traveler

Howard Barnum

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Abstract

I consider the “Quantum Bayesian” view of quantum theory as expounded in a 2006 paper of Caves, Fuchs, and Schack. I argue that one can accept a generally personalist, decision-theoretic view of probability, including probability as manifested in quantum physics, while nevertheless accepting that in some situations, including some in quantum physics, probabilities may in a useful sense be thought of as objectively correct. This includes situations in which the ascription of a quantum state should be thought of as objectively correct. I argue that this does not cause any *prima facie* objectionable sort of action at a distance, though it may involve adopting the attitude that certain dispositional properties of things are not “localized” at those things. Whether this insouciant view of nonlocality and objectivity can survive more detailed analysis is a matter for further investigation.

1 Introduction

Quantum states are the heart of the predictive and explanatory power of quantum physics. On any interpretation of quantum theory, the quantum state (wavefunction or, more generally, density operator) of a system is a tool for calculating the probabilities of whatever future measurements we may choose to make on the system. In fact, the quantum state can be considered to be a succinct *representation* of the probabilities for the outcomes of any possible measurement on the system. In this paper, I consider interpretations of quantum theory on which this is the *essence* of quantum states, and focus on the nature, objectivity, and reality of the probability ascriptions represented by quantum states. Since I subscribe to a broadly Bayesian, personalist account of probability insofar as it relates to the world, my analysis will have to make sense of quantum probabilities in personalist Bayesian terms. But the main concern of the paper will be to argue that a notion of objective probability—“propensity”, if you like—can be compatible with the personalist view, and that quantum probabilities sometimes have this nature.

More specifically, while I agree with the “Quantum Bayesian” or “QBist” perspective of Carlton Caves, Christopher Fuchs, and Rüdiger Schack as expressed in [3] on many points, in this paper I will take—as have others—a position which some (including

CFS, an acronym which I will use to refer to Caves, Fuchs, and Schack as authors of [3]; some of their views may have changed since then) might consider more “straight Copenhagen”. The position is that in at least some situations an ascription of a quantum state to a system can be “objectively correct” in light of quantum theory, general background knowledge and common sense, and specifics concerning the system in question (how it was prepared, the outcomes of previous measurements made on it, and so forth).

Having argued for this claim, I will then examine what it implies about the nature of reality, and of the quantum state’s relation to it, especially in light of the arguments of EPR and the evident concern of the “QBists” about a link between objective probabilities and objectionable nonlocal effects. I will argue it does not support any claim that the quantum state is a real *entity*, and although it may sometimes commit us to ascribing dispositional or modal *properties* to systems, which are perhaps real in the weak sense that systems *really have them*, it is far from clear that we must view these modal properties as *located in* the system, or that their nonlocal change when the state of a system “collapses” when a measurement result is obtained on its entangled partner system, should bother us.

2 Personalist probability and quantum states

2.1 This I believe: personalist Bayesianism is the essence of applied probability

Let me re-emphasize that I view probabilities that occur in physical theories as ultimately guides to action. They determine rational betting odds, if you like. Personalist, decision-theoretic accounts of probability, along the lines of Bruno de Finetti, Leonard Savage [8], and more recently, Richard Jeffreys [6], are just fine with me. The point of including Jeffreys is to emphasize that I’m not wedded to *conditioning* on newly acquired certain knowledge as the only reasonable way in probabilities can change in light of our interactions with the world, a point on which I agree with CFS. Rather, maintaining *coherence* of one’s probabilistic beliefs in the light of continued interactions is the goal. Let me also emphasize—for it will be important—that I view *scientific progress* itself as best understood—partly in the sense of “rational reconstruction” but also to a large extent as regards actual, if often implicit, practice (cf. [6])—in terms of such Bayesian (broadly-speaking) updating of beliefs. As an undergraduate studying philosophy at a time when, and in a department where, Nelson Goodman’s *Fact, Fiction, and Forecast* [5] and Quine’s “Two dogmas of empiricism” [7] were iconic reference points, I took the lesson of Goodman’s paradox of

the grue emeralds to be the inescapable need for prior beliefs about the plausibility of alternative hypotheses compatible with the data, in order to get any kind of inductive reasoning off the ground, and soon adopted a generally Bayesian analysis of “inductive reasoning”. The Duhem-Quine thesis that our beliefs meet the world as a body, a linked web which may adjust to empirical experience in various ways also formed part of this view. This makes me suspicious of views which rely too heavily on “foundational” approaches to knowledge and scientific progress in our corpus of beliefs contains a sharply delineated foundation of “certain knowledge”, for example the “protokolsatze” of some of the early Vienna School positivists. Still, within the “web of belief” some statements may be closer to our immediate experience than others, so I am wary of adopting a too-glib anti-foundationalism as well. A sympathetic view of the Quine-Duhem thesis also disposes me toward the Richard Jeffreysian view of belief change while maintaining probabilistic coherence which Fuchs and Schack have recently recommended to me. This broadly “Bayesian” viewpoint is still at the core of my views on knowledge and how we acquire it.

2.2 Quantum probabilities, personalist and objective

It might seem surprising, then, that I would favor the view that quantum probabilities can be, in important situations, objective. This should not, however, be so surprising: by “objective” I do not mean that they are not to be understood in personalist, Bayesian terms as “guides to action”, nor do I mean they are facts about the frequencies of certain types of events in a “block world” in which all events, past and future, are laid out, described, and classified. I mean that in the sense in which the fact that the Sun will rise tomorrow is an *objective fact* of physical and astronomical science, I’d grant objectivity to some quantum probabilities. That the Sun will rise tomorrow is a proposition one could logically disbelieve; no evidence we currently have *logically compels* one to believe it, and one could presumably come up with priors under which the currently available evidence would still leave one in significant doubt about it. It is, in the end, a personalist probability judgement of near-certainty for a particular event—a judgement that virtually all of us agree on, and a judgement that we view as objectively correct. It is in this sense that I want to maintain that quantum states can be objectively correct, as a matter of quantum physics plus background knowledge. While at some deeper level it—like all judgements—may be in some sense personalist and subjective, I believe that for one who accepts quantum physics, plus various uncontroversial statements about observable (“classical” if you like) facts concerning experimental setups, there are situations in which there’s a particular state vector or density matrix that represents the correct beliefs, and that should guide one’s future actions, concerning this quantum system.

The view I defend here may be generally identified with what CFS call “Copenhagen-

like interpretations” of quantum mechanics, but there is one possibly critical difference. CFS say “To our knowledge, these [“realist readings of the Copenhagen interpretation”]¹ all have in common that a system’s quantum state is determined by a sufficiently detailed, agent-independent *classical description* of the preparation device, which is itself thought of as an agent-independent physical system.” I would not want to claim that this holds in all situations. The terminology “the preparation device” suggests that CFS only intend it to hold when a system is viewed as “having been prepared”, but I’m not sure I’d even go that far. Nevertheless, I’d claim, perhaps contra CFS, that in many such preparation situations, the quantum state is determined (not in the causal sense) by “classical” facts about preparation devices. I don’t necessarily take “classical” in the sense of “classical physics”; “ordinary-language” might be closer. But the description could contain technical, theory-laden terms, like “polarizing beamsplitter”; see below.

It’s not always clear to me that this represents a serious divergence from CFS’ views, rather than a matter of emphasis.

It’s important here to separate, at least *a priori*, the issues of the *objectivity* of quantum states, from issues of their *reality* or ontological status. The term “objectivity” with its root of “object” suggests an assignment of ontological status to “objective probabilities” which I’d like the option of avoiding unless it becomes clear that it’s compulsory.

Another point that is important to CFS is the agent-dependence of quantum states. Fuchs and Schack like to emphasize that the quantum state is relative to a specification of an agent and system. When I do an experiment on X , X has a wavefunction for me (or I have one for it). On a more objective-probabilities view quantum theory sometimes tells me what my wavefunction should be. But this could, if needed, be understood as advice for a certain agent, contemplating a certain thing “as quantum”. I am willing to accept a degree of agent-dependence—or, perhaps, background-information dependence—of quantum probabilities even in cases when I believe there may be objective correctness to these probability assignments.

In my view, “objective correctness” is not exactly a matter of rejecting the notion that these probabilities “always depend on a prior.” Rather I believe that just about everything in science and everyday life “depends on a prior” if “rationally reconstructed”; but that in many things, there is sufficient agreement that we have converged on subjective probabilities that they may as well be granted the term “objective”. In particular, even the everyday-life assertions of fact that are taken as unproblematic, I take to have something like this character.

¹By “realist readings” I take CFS to mean readings that are realist about “the world”, not necessarily realist about the quantum state

Perhaps my notion of objective probabilities is so subjective that, CFS would be willing to endorse it. In this case, one wonders why they go to such great lengths to emphasize the subjective nature of quantum probabilities. One likely reason is that they wish to make it clear that they do not accept the notion of an “objectively real” quantum state as physical object, of the sort that appears in most versions of the de Broglie-Bohm and Everett interpretations. But I worry that they are underestimating the extent to which we can have both all the objectivity anyone could reasonably want for at least some quantum state ascriptions—a degree of objectivity which, moreover, it seems to me would be unreasonable not to grant in some cases—while also not *reifying* the quantum state as an object out in the world, and especially, not as an object localized at the system whose state it is. I believe that an unfounded fear that these two features of an interpretation might be incompatible, and of the “action at a distance” that would follow from dropping the latter feature and allowing quantum states as physical objects in the world, has led CFS to project an excessively subjectivist view of the nature of quantum theory, and to unnecessarily differentiate their views from that part of the Copenhagen tradition—represented in our time by, for example, Mermin, Brukner and Zeilinger, and earlier by Peierls and perhaps even Bohr—that views the quantum state as essentially epistemic, in the sense of representing our best guide to action in the face of uncertainty about the outcomes of alternative possible measurements.

There is more to it, however: I believe that CFS at different places adopt two versions of the claim that quantum theory never determines state ascriptions, corresponding to two different interpretations of the words “quantum theory” in this claim. The stronger version, which I take exception to, claims that “quantum physics” broadly construed never determines state ascriptions; the weaker one, which seems obviously correct, claims that “the quantum formalism” of density operators and positive operator valued measures does not determine it. There are discussed at more length in the next section, and the implications of the distinction for the QBist program, and related programs viewing the quantum state as epistemic, are discussed to in the conclusion.

3 Meaty quantum physics versus *théorie quantique minceur*

Before going into more detail about objective state assignments and state preparation, I make a distinction which bears importantly on the meaning of “Quantum Bayesianism” and on the nature of the QBist project (as I imperfectly understand it).

The distinction is between *quantum physics* and what might be called *the quantum formalism*, or more specifically, the Hilbert space/POVM formalism. The quantum formalism is what is sometimes called “quantum probability” (not to be confused with notions of not-necessarily-positive real or complex-valued probabilities that sometimes go by this name): a particular generalization or “extension” of the formal apparatus of classical Kolmogorovian probability theory, from among the many mathematically consistent ways of extending this theory to allow for the possibility of choosing alternative, incompatible measurements (or equivalently, incompatible random variables). Such extensions model a system by specifying a compact convex set of possible normalized system states, which can serve as the base for a cone of unnormalized states, and a compact convex set of possible measurement outcomes that is an initial interval in the cone dual to the cone of states (or at least, in a subcone of this)². There is nothing nonstandard about the probabilities that occur in such models; mathematically, they are completely standard and therefore the states in such models are perfectly susceptible to Bayesian interpretation as guides to betting on the outcomes of measurements. The quantum formalism, the formalism of Hilbert space density matrices (the normalized states) and POVM elements (the initial interval in the dual cone), is a specific case of this general formalism. The principle that the states one can prepare, and the measurement outcomes one can obtain, for a system, can be modeled within this quantum formalism is indeed, therefore, “an empirical addition to Bayesian coherence”. I’d add only that the general framework of convex state and effect spaces—as further developed to include dynamics (both unconditional and conditional on measurement results), can be understood as representing “Bayesian coherence” in settings where alternative, “incompatible” (in a sense that can be made precise within the convex sets framework) measurements may be made on a given system.

Quantum physics, on the other hand, is the full, meaty physical theory, in which the states in particular orthonormal bases carry physical significant labels, and we truck with notions of forces, particles, interaction Hamiltonians, fields, and so forth.

The line between meaty quantum physics and the math-y quantum formalism (the “théorie quantique minceur” of the section title) is somewhat blurry. Preferred tensor factorizations, observables, symmetries might help make the transition between the two. Indeed, symmetries may lead us to consider convex state spaces other than the most standard quantum ones, for instance through Abelian or nonabelian superselection rules.

An important point is that the quantum formalism may be viewed as a *representation* of probabilities, for example for particular preparation procedures followed

²See, for example, [4] or the introductory sections of [1, 2], among many possible references, for introductions to this formalism.

by particular results of particular measurement procedures—arrived at through the usual process of scientific inference, which I view as in essence Bayesian. Such actual preparation and measurement procedures contain plenty of meaty detail in their description. A formal probabilistic structure can be abstracted from them—for example, by the time-honored strategy of lumping together as “preparing the same state” all preparation procedures that lead to the same list of probabilities of the various measurement outcomes’ occurring, and lumping together as “the same effect” all measurement outcomes that give rise to the same list of probabilities of occurring in the various states. But it should not be forgotten that this structure has its roots in such experimental reality, and thus, indeed, in the very same inferences that led us to adopt the full “meaty”, “physics-y” quantum theory itself.

Statements like that of Fuchs and Schack, that “the basis for one’s particular quantum state-assignment is always *outside* the formal apparatus of quantum mechanics” suggest that they may only be concerned to assert subjectivity relative to “the quantum formalism”, and not to “meaty quantum theory” as I’ve described it above. This would provide a way out of the need to deny the “objectivity” of quantum state ascriptions such as that of the state prepared by a polarizing beamsplitter, etc... These state-ascriptions are objective according to *quantum physics*, but are certainly not determined by the mere *quantum formalism*. I am not sure if this is really the route CFS, or at least Fuchs and Schack, want to take; if so, I think we are close to complete agreement, but in [3] there still seems a push to deny the objectivity even of state assignments warranted by full, meaty applied quantum physics.

4 State preparation

Let’s take an example where quantum probabilities may reasonably be thought to be objective. Send a single photon through a polarizing beamsplitter and do high-efficiency photodetection at one output. When the photon passes without a count, *quantum physics* plus general background knowledge determines the probabilities of counts at the outputs of a second photodetector, perhaps differently oriented, placed after the free output of the first—i.e. the state of the polarization degree of freedom.

4.1 Quantum description of the measurement device

CFS argue against the contention that such a probability assignment may be objective on the grounds that even in cases of state preparation, the fact that the preparation device carries out the claimed state preparation operation, depends on subjective beliefs summarized in the quantum state of the preparation apparatus.

I agree that *if one does* give a quantum description of the measuring device, the operation it performs will be dependent upon one's assignment of a quantum state to it—one's beliefs about it. But who says a quantum analysis of the apparatus is needed? From a point of view from which what we take to be quantum depends upon our purposes and the experiments we plan to do or other projects we plan to undertake, to suddenly *require* that we do a quantum analysis of the apparatus seems to put us in danger of becoming unwitting victims of the cult of the larger Hilbert space. We don't need a weatherman to say which way the wind blows, and we don't need a full quantum analysis of something to tell us it's a polarizing beamsplitter, though it is nice that a quantum analysis jives with a more everyday story. This observation is somewhat analogous to the observation that we don't need neuroscience to (usually) trust in our perceptions, except that “polarizing beamsplitter” is certainly more theory-laden than many terms used in reporting perceptions. I'd argue that this theory-ladenness does not invalidate the claim that the probabilistic description of the operations performed by such devices can be viewed as developed to some extent at the phenomenological level. Indeed, these descriptions were developed as part of the process that created quantum theory *as a theory that gets its hooks into the physical world*. In this process we already made inferences to—ultimately personalistic, but about as objective as anything gets in physical theories—probabilistic statements about interactions between systems like that and measuring devices like this. These inferences were not themselves based on full-fledged quantum theory, nor on explicit prior quantum state assignments to measurement apparatus. Rather, there was a simultaneous development of concepts like “photon”, “photon polarization”, and “polarizing beamsplitter.” If, upon making the optional but, it might be argued, obligatory-that-it-be-in-principle-possible-to-make, transition to a *quantum* view of the measuring apparatus (or part of it), this commits us to quantum state ascriptions, perhaps that is all right. That is, after all, part of what I've been arguing is part of a *meaty* (or if you prefer, *tofu-y* or *texturized-vegetable-proteiny*) version of quantum physics, which is the kind we've actually got. Of course in the end this probably won't commit us to a pure state ascription, but only to specifying some very coarse-grained features of the quantum state of the apparatus that are sufficient for it to perform the operation it does. This might be reflected in assigning a highly mixed density operator with certain properties—or even just asserting certain facts about the apparatus density operator, without necessarily reflecting further on the exact density operator we choose within the subspace of satisfactory ones.

4.2 Estimating the behavior of apparatus

Another important point is that we can do statistical estimation of the relevant facts about how our apparatus works (and experimenters often do), starting from an exchangeable prior for repeated use of the beamsplitter that's not necessarily based on

quantum theory.³ Yes, as CFS point out, this involves a prior; but many of our priors, such as the likely exchangeability of certain sequences of events, reach beyond the specifics of quantum theory into the way we think everyday objects and situations behave. Experimental apparatus can be defective from time to time, or badly designed, and in some cutting-edge situations there may be real questions about whether quantum effects critical to its correct operation have been analyzed correctly—but it would take some pretty kooky beliefs about quantum states (or less outlandishly, some far-fetched paranoia about the folks at the optics company) to lead to nonstandard beliefs about a polarizing beamsplitter that would survive a few basic checks.

5 Elements of reality?

I've argued that sometimes it is an objective fact, in light of quantum physics and other knowledge, what the probabilities of certain measurement outcomes would be should we make the relevant measurement. Does that make these probabilities *elements of reality*?

EPR famously said “yes”, at least in the case when a measurement outcome will give probability 1. It's worth including the passage:

“A comprehensive definition of reality is, however, unnecessary for our purposes. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* [...] Regarded not as a necessary, but merely as a sufficient, criterion of reality, this criterion is in agreement with classical as well as quantum ideas of reality.”

However, I suspect that probabilities can be objective without being elements of reality.

Here's how I'd like to view the situation. Quantum theory *recommends*—or *insists on* certain constraints on decisionmaking, in some particular situations in which an agent knows a certain measurement will be performed (or measurement-like situation will arise), and has particular background knowledge. Perhaps one might want to

³ An experimenter's implicit prior about the performance of an apparatus is likely to be much more complicated than just straight exchangeable ones—in particular, for a single complex experimental setup used many times in succession, they are likely to include a reasonable weight on the possibility of systematic, time-correlated drift due to environmental conditions in the lab; tighter correlations in a single “run” of use of the setup stemming from the effects of initializing the apparatus, and so forth. Such considerations don't necessarily vitiate my basic point.

say the quantum state is “law-like”. What I’d really like to say, though, is that this fact about how one should bet describes a *dispositional property* of the system—it is disposed to give a certain outcome, if a particular measurement is performed. But is this property *real*? Well, what does that even mean? If we mean “does the system really have the property?”, well, sure. But does the property *exist*? What are some analogies that would help us understand what this means. Is it like the roughness of a rough rock? I guess we could say the roughness of the rock exists (I’m not sure how compelling that is) and seems to be located at the rock. But the roughness is a *property*, not an object. Does that matter? Well, even if we’re led to say it’s “real”, that won’t make it an *object* as it is in the Everett interpretation. How would we treat objective probabilities in a classical theory whose basic equations were stochastic equations describing, say, the movements of particles? Again, I think we’d say they are lawlike statements about the dispositions of particles; we wouldn’t necessarily say the transition probabilities were themselves an existent field.

Of course, some versions of Bohmianism say the quantum state (“guiding wave”) is lawlike, rather than object-like. My version of the Copenhagen interpretation, though, isn’t Bohmian for other reasons: the probabilities are taken to concern, not underlying ontic “positions” and “momenta” but measurement outcomes. Of course, these outcomes may be taken to include outcomes *of position measurements*. But there are other observables that can be measured as well, and the theory is silent on what’s measured where.

Well, say we take it as a real property. Should we be bothered by the prospect of its nonlocal change?

Are such properties even **at the system**?

Should EPR bother us? Instantaneous change of a state of knowledge needn’t bother us. Instantaneous communication would bother us. But this doesn’t permit it. Let’s compare this to the real property of, say, roughness. We would be more worried if we could change something from rough to smooth instantaneously at a distance, because that could be used to signal. The fact that the change in properties of *B* represented by collapse in the EPR scenario *cannot* be used to signal seems to be a deep difference between such properties and more commonplace ones like roughness. Is it a manifestation of the intrinsically relational, intrinsically less localized, nature of such quantum properties? I guess I’d say so. (Or should we instead take this as evidence that they are “less real”. Again I wonder what would be the point of using, or not using, this form of words. It is probably better to attend to how the property actually behaves, compared to e.g. roughness.)

We seem on solid ground if we wish to maintain that the fact about how Alice should bet is not a fact about *how things are at Bob’s site*. It’s not reasonable to say

that quantum physics recommends that Bob should immediately change his betting behavior to the one it recommends for Alice. (Though I suppose it does claim that he'd be better off if he did.)

The determinants of states can be “unproblematic”, macro-facts. E.g. in EPR, Alice’s measurement results, even if relativistic causality prevents them being known to Bob for awhile.

5.1 Instrumentalism, reality, and the quantum state

The quantum state is certainly a useful instrument even if it isn’t a real property of the system. Emphasizing that a *correct, objective, or fact-like* quantum state is a correct (objective, fact-like) prescription for betting—a good guide to action—stresses this useful, instrumental nature of the state. Does this give reason to doubt that the such objective quantum probabilities are a “real properties” of the world?

It can be an *objective fact* which instrument is useful in a given situation. But also, the fact that a certain instrument is right to use in a given situation tells us something about how things really are in that situation. So an instrumental reading of the quantum state doesn’t necessarily get us out of viewing quantum probabilities as “real properties” (or in more EPR-like language, “corresponding to” real properties). Indeed, maybe more facts than we think about reality are of this nature—enough so that we’ll want to call $|\psi\rangle$ real after all?.

“That’s a table in front of me” vs. “reality is such that I’m well-advised to use the concept of table, with all the predictive (it will support things) and retrodictive (somebody made it) baggage it brings along, in dealing with this particular situation.” Not much difference, is there? Words as tools, as the cartoon version of Wittgenstein goes. Facts are stated using words. To state a fact is to use a tool?

Could the counterfactual (dispositional) nature of the property “will yield spin up in a measurement of σ_x ” save it from being real? But one might argue all, or many, concepts involve such dispositional aspects.

Indeed, the notion of reality might seem to *require* some counterfactual thinking. Does it necessarily describe agents who manipulate, interact? Hacking on realism about entities: “if you can spray them, they are real”. We are, of course, more concerned with properties. Is *sprayability* real?

On balance, I’m inclined to think of quantum states, in situations when quantum physics broadly construed dictates the state, as real properties of the world. I’m really not sure what “real” adds to “property” in this statement, though. Should we think of

them as real properties of the systems whose states they are? Again, what does “real” add? Does it mean they are *not relational*, i.e. they are independent of properties of other systems? In this case—depending on what is meant by “independence”—the property may not be real. If “independent” means that it only concerns what will happen in future measurements on the system, then perhaps the property is not relational. Of course, realizing the counterfactual involves bringing it into relation with another system, the measuring system. But as a disposition to behave a certain way if such relations are brought about, it is not itself relational. On the other hand, we may, as in the EPR case, *ascribe* the property because of relations to other systems—so in *that* sense it may need to be thought of as relational.

There is no decisive test one can perform *on a system*, B , to determine whether B is, or is not, in state $|\psi\rangle$. There are tests—measurements in a basis containing ψ —that can provide some confirmation for this state ascription, and that can falsify the state ascription (but are not guaranteed too even if “it’s wrong”). But positive grounds for definitely believing that the system *now* (at, say, time t) has the property of having state $|\psi\rangle$ cannot be had by further measurements (at times $t \geq t'$) solely on system B . This is a major way in which such quantum “properties” differ radically from classical ones, and, perhaps, a reason why their change “at distance” via collapse, should be viewed with less alarm than a similar classical change. Indeed, this very nonclassical inability to locally, retrospectively verify such properties makes them quite different from classical properties such as roughness and, significantly, is necessary in order to avoid instantaneous signalling.

The way the world *has* responded to Alice’s probing at her site, is correlated with the way the world *will* respond at Bob’s site. We can’t use this to signal. What’s the problem? If these “laws of response” are real, then this aspect of reality is not localized, and that’s about all one can say. What’s the problem?

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